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FRW Cosmological Models with Bulk-Viscosity in Barber's Second Self-Creation Theory

S.D. Katore · R.S. Rane · K.S. Wankhade

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Abstract We have studied the evolution of spatially homogeneous and isotropic FRW cosmological model with bulk-viscosity in the frame work of Barber's (Gen. Relativ. Gravit. 14: 117, 1982) second self-creation theory of gravitation. The cosmological models are obtained with the help of special law of variation for Hubble parameter proposed by Bermann (Nuovo Cimento 74B: 182, 1983). Physical parameters of the models have been discussed in case of false vacuum model, Zel'dovich fluid and radiation dominated fluid.

Keywords FRW cosmological model · Bulk-viscosity · Hubble parameter · Barber's self-creation theory

1 Introduction

In recent years, there has been a considerable interest in alternative theories of gravitation. Brans-Dicke [4] theory develops Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution of matter in motion. Brans-Dicke theory is a scalar-tensor theory of gravitation in which the tensor field is identified with the space-time of Riemannian geometry and scalar field is alien to geometry. This theory does not allowed the scalar field to interact with fundamental principles and photons. However, Barber [1] has modified scalar-tensor Brans-Dicke theory to develop a continuous creation of matter in the large scale structure of the universe. As a result, two self-creation theories are proposed

S.D. Katore
PG Department of Mathematics, SGB Amravati University, Amravati, India
e-mail: katoresd@rediffmail.com

R.S. Rane · K.S. Wankhade (✉)
Department of Mathematics, Y. C. Science and Arts College, Mangrulpir, India
e-mail: wankhade.kishor@rediffmail.com

R.S. Rane
e-mail: rsrane53@rediffmail.com

by Barber [1] in which the mass of the universe is seem to be created out of self-contained gravitational, scalar and matter field. Brans [3] has pointed out that Barber's first theory is in disagreement with experiment as well as inconsistent, in general. Hence Barber's first theory is not accepted because this theory violets the equivalence principle.

The second theory retains the attractive features of the first theory and overcomes previous objections. These modified theories create the universe out of self-contained gravitational and matter fields. In Barber's second self-creation theory, the gravitational coupling of the Einstein field equations is allowed to be a variable scalar on the space-time manifold. Barber's second theory is a modification of general relativity to include continuous creation and is within observational limits, thus it modifies general relativity to become a variable G-theory. In this theory the scalar field does not directly gravitate, but simply divides the matter tensor, with the scalar acting as a reciprocal gravitational constant. The scalar field is postulated to couple to the trace of the energy-momentum tensor. The consistency of Barber's second theory motivates us to study cosmological model in this theory.

Many authors have studied various aspects of Robertson-Walker model in Barber's self-creation theory of gravitation to produce mass creation in presence of perfect fluid satisfying the equation of state $p = (\gamma - 1)\rho$, where $1 \leq \gamma \leq 2$. Pimentel [9] and Soleng [23] have discussed FRW models by using a power law relation between the expansion factor of the universe and the scalar field. Singh [22], Reddy [15] and Reddy et al. [16] have studied Bianchi type space-times solutions in Barber's second theory of gravitation while Reddy and Venkateswarlu [17] presented Bianchi type—VI_0 cosmological model in Barber's second self-creation theory of gravitation. Shanti and Rao [24] studied Bianchi type II and III space-times in this theory, both in vacuum as well as in presence of stiff fluid. Ram and Singh [14] have discussed the spatially homogeneous and isotropic Robertson-Walker and Bianchi type-II models of the universe in Barber's self-creation theory in presence of perfect fluid by using gamma law equation of state. Pradhan and Pandey [11], Pradhan and Vishwakarma [12], Panigrahi and Sahu [10], Vishwakarma and Narlikar [26], Sahu and Mohanty [19], Singh and Kumar [20], Singh et al. [21] and Venkateswarlu et al. [25] are some of the authors who have studied various aspects of different cosmological models in Barber's second self-creation theory. In recent years, Katore et al. [6], Reddy and Naidu [18] and Pradhan et al. [13] have studied the cosmological models with deceleration parameter and constant deceleration parameter model of the universe in the context of different aspects of different space-time.

Bulk viscosity is supposed to play a very important role in the early evolution of the universe. There are many circumstances during the evolution of the universe in which bulk viscosity could arise. Further, there are several aspects which are expected to study the effect of bulk viscosity. These are the decoupling of neutrinos during the radiation era and the decoupling of matter and radiation during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. It is known that the introduction of bulk viscosity can avoid the big bang singularity. Pradhan and Pandey [11] proposed Barber's second self-creation theory with bulk viscous fluid source for an LRS Bianchi type-I space time by using constant deceleration parameter where the metric potentials are taken as function of x and t.

In this paper, we have investigated spatially homogeneous and isotropic FRW cosmological model in presence of bulk viscous fluid within the framework of Barber's second self-creation theory of gravitation. We have obtained an exact solutions of field equations of Barber [1] by taking the law of variation of Hubble parameter proposed by Bermann [2] that yields an accelerating model of the universe. While doing so, we have used the equation of state $p = (\gamma - 1)\rho$ where $(0 \leq \gamma \leq 2)$ is a constant. The physical behavior of the models such as the Zel'dovich fluid, false vacuum and radiation are discussed.

2 Metric and Field Equations

We consider the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) line element in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{1}$$

where $R(t)$ is the scale factor, K is the curvature parameter which can take the values $K = 0, -1, +1$ for flat, open and closed universe respectively and the signature of the metric is $(+, -, -, -)$. The field equation in Barber's second self-creation theory of gravitation are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\varphi^{-1}T_{ij} \tag{2}$$

and the scalar field equation is defined by

$$\Box\varphi = \frac{8\pi}{3}\lambda T, \tag{3}$$

where $\Box\varphi = \varphi_{;k}^{\ k}$ is the invariant d'Alembertian and T is the trace of the energy-momentum tensor that describes all non-gravitational and non-scalar field theory. Barber scalar field φ is a function of t due to the nature of space-time which plays the role analogous to the reciprocal of Newtonian gravitational constant i.e. $\varphi = \frac{1}{G}$. λ is a coupling constant to be determined from the experiment $|\lambda| < 10^{-1}$. In the limit as $\lambda \to 0$, this theory approaches the standard general relativity theory in every respect.

We assume that the fluid has bulk-viscosity, so that the energy-momentum tensor can be written as

$$T_{ij} = (\rho + \bar{p})U_i U^j - \bar{p}g_{ij} \tag{4}$$

together with

$$U_i U^j = 1 \tag{5}$$

and

$$\bar{p} = p - \eta U_{;i}^{\ i}. \tag{6}$$

Here ρ is the energy density, p is the isotropic pressure, $\bar{p}$ is the effective pressure, η is the bulk viscosity coefficient and U^i is the four-velocity vector of the matter distribution defined by $U^i = \delta_4^i$, $i = 1, 2, 3, 4$. We assume that the coordinate to be commoving so that $U^i = (0, 0, 0, 1)$. Since the bulk viscous pressure represents only a small correction to the thermo-dynamical pressure, it is a reasonable assumption that the inclusion of viscous term in the energy-momentum tensor does not change fundamentally the dynamics of the cosmic evolution. In general, η is a function of time. A comma (,) and a semi-colon (;) denotes ordinary and covariant differentiation respectively. For a universe field with bulk viscous fluid, form (2) one finds

$$T_1^1 = T_2^2 = T_3^3 = -\bar{p}, \qquad T_4^4 = p \quad \text{and} \quad T = \rho - 3\bar{p}. \tag{7}$$

In a commoving coordinate system the Barber's field equations (2) and (3) for the metric (1) with the help of (7) take the form

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{K}{R^2}=-8\pi\varphi^{-1}\bar{p}, \tag{8}$$

$$3\frac{\dot{R}^2}{R^2}+\frac{3K}{R^2}=8\pi\varphi^{-1}\rho, \tag{9}$$

$$\ddot{\varphi}+3\dot{\varphi}\frac{\dot{R}}{R}=\frac{8\pi\lambda}{3}(\rho-3\bar{p}) \tag{10}$$

and

$$\bar{p}=p-3\eta H, \tag{11}$$

where $H=\frac{\dot{R}}{R}$ is the Hubble parameter, λ is a coupling constant to be determined from experiments ($|\lambda|\le 10^{-1}, \lambda=0$) and $G_N=\varphi^{-1}$. Equation (10) is the scalar field cosmological equation. Here overhead dots (.) indicate the differentiation with respect to t.

3 Solutions of the Field Equations

We are at liberty to make some assumption as we have more unknowns R, ρ, p, η and φ with lesser number of field equations (8)–(10) to determine them. For the complete determination of these field equations, we use a special law of variation of Hubble parameter, proposed by Bermann [2] as

$$H=DR^{-m}, \tag{12}$$

where H is the Hubble parameter defined by

$$H=\frac{\dot{R}}{R} \tag{13}$$

D and m ($\neq 0$) are constants.

If we use (12) and in (13), we obtain

$$R=[m(At+B)]^{\frac{1}{m}}, \tag{14}$$

where A and B are taken to be positive constant of integration.

The deceleration parameter is defined by

$$q=-\frac{\ddot{R}R}{\dot{R}^2}. \tag{15}$$

For the special law (14), (15) yields

$$q=m-1. \tag{16}$$

To obtain a determinate solution we also use a relation between scale factor $R(t)$ and a scalar field φ such that

$$\varphi(t)=\alpha R^n, \tag{17}$$

where α is a constant.

Using (14) and (17) in the field equations (8)–(10), we obtain

$$\varphi(t) = \alpha[m(At+B)]^{\frac{n}{m}}, \tag{18}$$

$$\bar{p} = -\frac{\alpha}{8\pi}\left[\frac{A^2(3-2m)}{\{m(At+B)\}^{2-\frac{n}{m}}} + \frac{K}{\{m(At+B)\}^{\frac{2-n}{m}}}\right] \tag{19}$$

and

$$\rho = \frac{3\alpha}{8\pi}\left[\frac{A^2}{\{m(At+B)\}^{2-\frac{n}{m}}} + \frac{K}{\{m(At+B)\}^{\frac{2-n}{m}}}\right]. \tag{20}$$

In order to solve (19) and (20), restricting the distribution with the barotropic equation of state i.e.

$$p = (\gamma - 1)\rho, \tag{21}$$

where γ is the adiabatic parameter. In general, the value of γ is taken to be constant satisfying the condition $0 \leq \gamma \leq 2$ and many authors have solved the field equations for different epochs by taking this constant of γ.

Now using (19) and (20), in (11) and (21), we obtain the explicit form of the physical quantities p and η as

$$p = (\gamma - 1)\frac{3\alpha}{8\pi}\left[\frac{A^2}{\{m(At+B)\}^{2-\frac{n}{m}}} + \frac{K}{\{m(At+B)\}^{\frac{2-n}{m}}}\right] \tag{22}$$

and

$$\eta = \frac{\alpha}{24\pi H}\left[\frac{3\gamma A^2 - 2m}{\{m(At+B)\}^{2-\frac{n}{m}}} + \frac{K(3\gamma-2)}{\{m(At+B)\}^{\frac{2-n}{m}}}\right]. \tag{23}$$

Thus FRW cosmological model by using (14) for interacting bulk-viscous fluid in second self-creation theory can be written (after suitable choice of coordinates and constants of integration ($m = 1$ and $B = 0$)) as

$$ds^2 = dt^2 - A^2t^2\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \tag{24}$$

and

$$p = \frac{3\alpha(\gamma-1)}{8\pi(At)^{2-n}}[A^2 + K], \tag{25}$$

$$\rho = \frac{3\alpha}{8\pi(At)^{2-n}}[A^2 + K], \tag{26}$$

$$\eta = \frac{\alpha}{24\pi H}\left[\frac{3\gamma A^2 - 2}{(At)^{2-n}} + \frac{K(3\gamma-2)}{(At)^{2-n}}\right] \tag{27}$$

and the deceleration parameter becomes

$$q = 0. \tag{28}$$

4 Physical Models

For complete discussion, we discuss three physical models corresponding to $\gamma = 0, 2, 4/3$ of the equation of state given by (21) for false vacuum model, Zel'dovich-fluid model and radiation model in Barber's second self-creation theory of gravitation.

4.1 False Vacuum Model (i.e. $\gamma = 0$)

For $\gamma = 0$, the distribution reduces to a special case with equation of state $\rho + p = 0$ which represents false vacuum or ρ vacuum [5]. The physical significance of this fluid in non-viscous anisotropic case has been studied by Mohanty and Pattanaik [7], while Mohanty and Pradhan [8] have investigated the viscous isotropic case.

In this case the physical quantities take the explicit form:

$$\rho = -p = \frac{3\alpha}{8\pi(At)^{2-n}}[A^2 + K] \tag{29}$$

and

$$\eta = \frac{-2\alpha}{24\pi H(At)^{2-n}}[1 + K]. \tag{30}$$

This model corresponds to a realistic physical situation when $\eta \geq 0$. This is true only when the Hubble parameter $H < 0$, but for flat space i.e. for $k = 0$ this situation is different.

4.2 Zel'dovich-Fluid Model (i.e. $\gamma = 2$)

For $\gamma = 2$, the distribution reduces to the equation of state $\rho = p$ which is called Zel'dovich fluid or stiff fluid. This equation of state is widely used in general relativity to obtain stellar and cosmological models for utter dense matter [27].

However in this case the density cum pressure and bulk viscosity coefficient take the form:

$$\rho = p = \frac{3\alpha}{8\pi(At)^{2-n}}[A^2 + K] \tag{31}$$

and

$$\eta = \frac{\alpha}{24\pi H(At)^{2-n}}[(6A^2 - 2) + 4K]. \tag{32}$$

4.3 Radiation Model (i.e. $\gamma = 4/3$)

For $\gamma = 4/3$, the equation of state is $\rho = 3p$, which represents the matter distribution with disordered radiation and a universe in which most of the energy density is in the form of radiation. In this case the physical quantities take the explicit form:

$$\rho = \frac{3\alpha}{8\pi(At)^{2-n}}[A^2 + K], \tag{33}$$

$$p = \frac{\alpha}{8\pi(At)^{2-n}}[A^2 + K] \tag{34}$$

and

$$\eta = \frac{\alpha}{24\pi H(At)^{2-n}}[(4A^2 - 2) + 2K]. \tag{35}$$

The solution obtained i.e., (33), (34) and (35) leads to an expanding model of the universe. It is interesting to note that the model is free from singularity.

5 Conclusions

In this paper, we have investigated spatially homogeneous isotropic FRW cosmological models in presence of bulk-viscosity within the frame work of Barber's second self-creation theory. To obtain determinate solutions of the Barber's field equations for FRW cosmological model, we have used a special law of variation of Hubble parameter proposed by Bermann [2]. It is observed that the introduction of bulk-viscosity avoids the occurrence of big-bang singularity. Clearly, this model is an expanding model of the universe. We have discussed the physical models corresponding to Zel'dovich fluid, false vacuum and radiation dominated fluid respectively. The model obtained in this paper is of considerable interest and may be useful in Barber's self-creation theory to study an accelerating model of the universe.

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